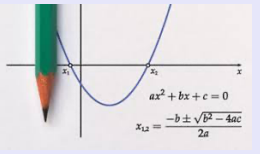
Math 125

Spring 2022

Lecture 26



Class QZ 19

Solve $2x(x-2)=1$ by the quadratic formula.

$2x^2-4x-1=0$

$ax^2+bx+c=0$

$a\neq 0$

$a=2$

$b=-4$

$c=-1$

$\Rightarrow b^2-4ac=(-4)^2-4(2)(-1)=16+8=24$ ✓

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{24}}{2(2)}=\frac{4\pm\sqrt{4}\sqrt{6}}{4}$

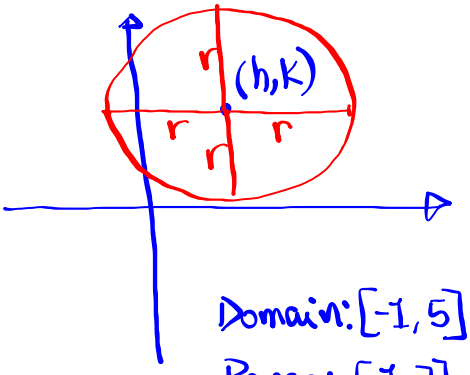
$=\frac{4\pm 2\sqrt{6}}{4}=\frac{2(2\pm\sqrt{6})}{4}=\frac{2\pm\sqrt{6}}{2}$ ✓✓

$\left\{\frac{2\pm\sqrt{6}}{2}\right\}$ ✓✓

Circle : $(x-h)^2+(y-k)^2=r^2$

Center (h,k)

Radius r



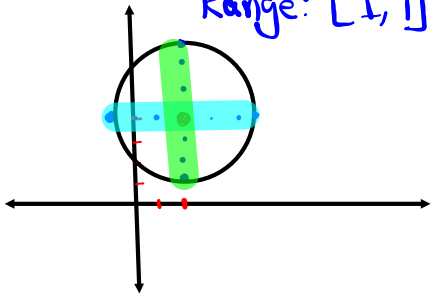
Ex: $(x-2)^2+(y-4)^2=9$

$(x-h)^2+(y-k)^2=r^2$

$h=2$, $k=4$ $r^2=9$

Center $(2,4)$

Radius $r=3$



Domain: $[-1,5]$

Range: $[1,7]$

Consider $(x+3)^2+(y-5)^2=25$

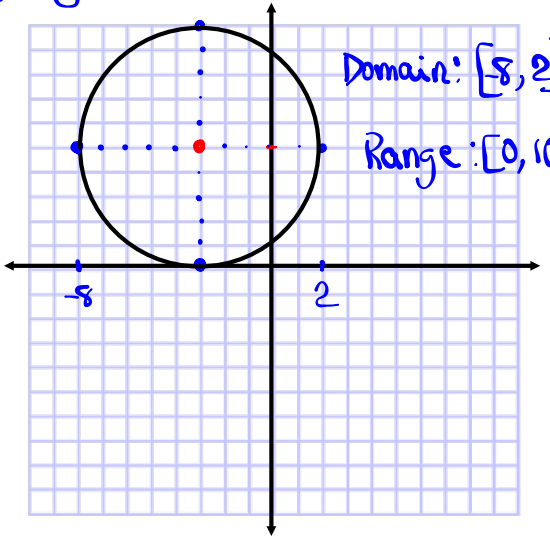
$(x-h)^2+(y-k)^2=r^2$

$h=-3$

$k=5$

Center $(-3,5)$

$r^2=25$ $r=5$



Domain: $[-8,2]$

Range: $[0,10]$

Your turn

$x^2 + (y+4)^2 = 16$

$(x-0)^2 + (y+4)^2 = 16$

$r^2 = 16$

$h = 0$

$K = -4$

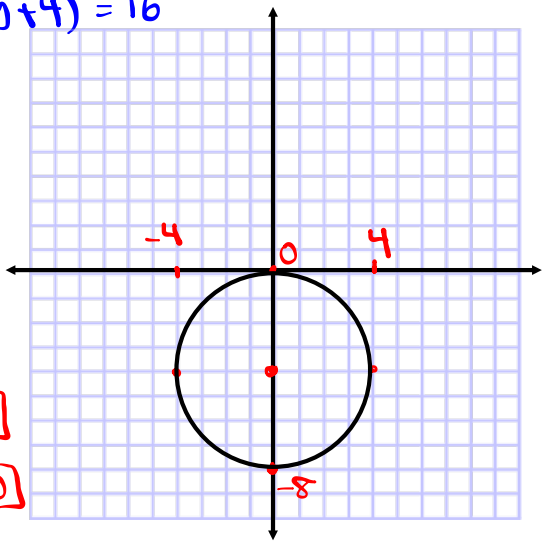
Center $(0, -4)$

Radius $r = 4$

Graph

Domain $[-4, 4]$

Range $[-8, 0]$



Your turn

$(x+5)^2 + y^2 = 16$

$(x+5)^2 + (y-0)^2 = 16$

$r^2 = 16$

$r = 4$

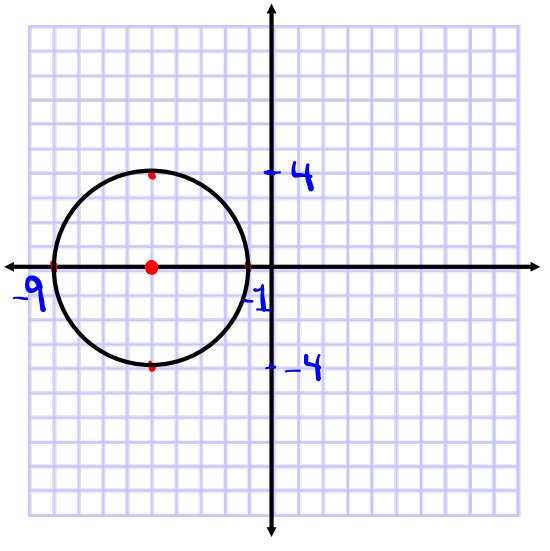
Center $(-5, 0)$

Radius $r = 4$

Draw

Domain $[-9, -1]$

Range $[-4, 4]$



Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (+ must, = must)

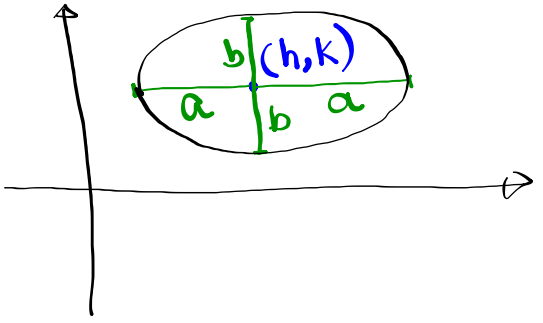
$a>0$

$b>0$

Center (h,k)

From the Center $\pm a$ units right & Left

From the Center $\pm b$ units up & down



$\frac{(x-5)^2}{4} + \frac{(y-6)^2}{16} = 1$

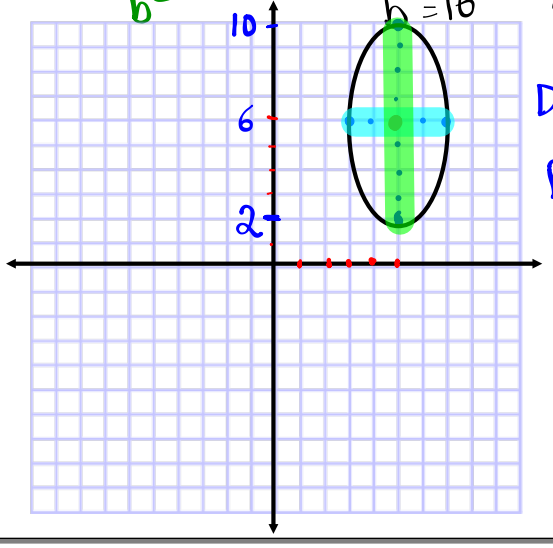
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$h=5$ $K=6$

Center $(5,6)$

$a^2=4$ $a=2$

$b^2=16$ $b=4$



Domain $[3,7]$

Range $[2,10]$

$\frac{(x+3)^2}{9} + \frac{(y-5)^2}{25} = 1$

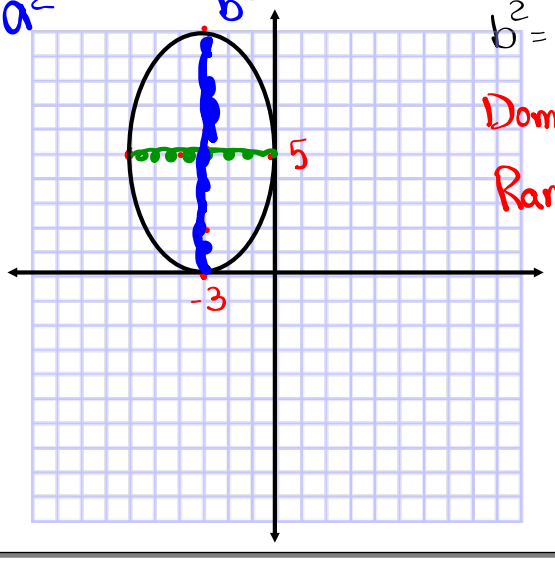
$h = -3 \quad k = 5$

Center $(-3, 5)$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$a^2 = 9 \quad a = 3$

$b^2 = 25 \quad b = 5$



Domain: [-6, 0]

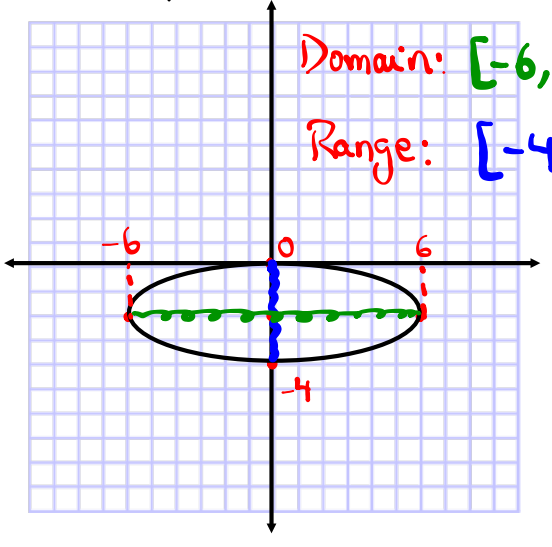
Range: [0, 10]

Your turn $\frac{x^2}{36} + \frac{(y+2)^2}{4} = 1$

Center $(0, -2)$

$a^2 = 36 \quad a = 6$

$b^2 = 4 \quad b = 2$



Domain: [-6, 6]

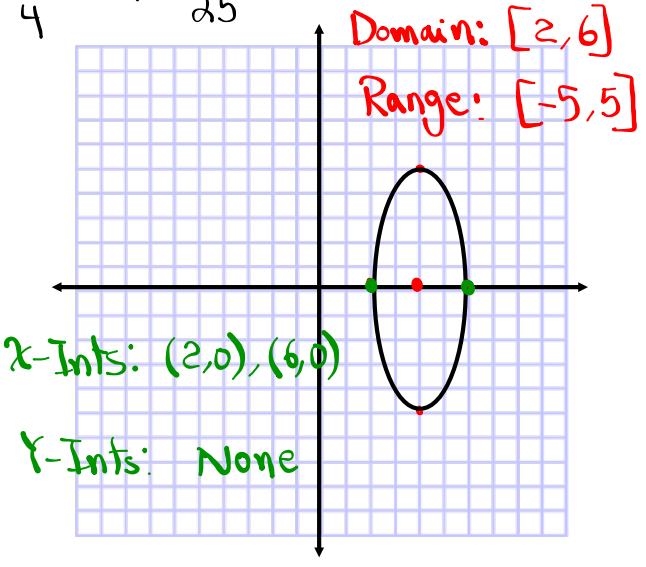
Range: [-4, 0]

Your turn: $\frac{(x-4)^2}{4} + \frac{y^2}{25} = 1$

Center (4,0)

$a^2 = 4 \quad a = 2$

$b^2 = 25 \quad b = 5$



$4x^2 + 25y^2 = 100$

$\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$

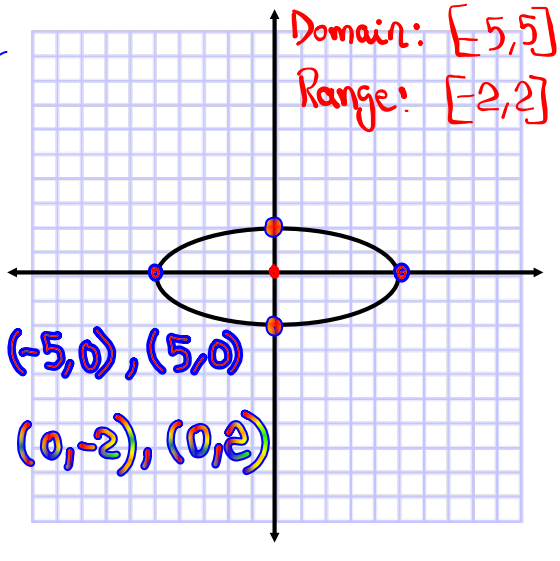
$\frac{x^2}{25} + \frac{y^2}{4} = 1$ ✓

Hint: RHS must be 1

Center (0,0)

$a = 5$

$b = 2$



Solve $(x^2-5)^2 - 2(x^2-5) - 15 = 0$

Hint: Let $u = x^2-5$

$u^2 - 2u - 15 = 0$

$(u-5)(u+3) = 0$

$u-5=0$ $\quad u+3=0$

$u=5$ $\quad u=-3$

when $u=5$

$x^2-5=5$

$x^2=10$

$x=\pm\sqrt{10}$

when $u=-3$

$x^2-5=-3$

$x^2=2$

$x=\pm\sqrt{2}$

$\{\pm\sqrt{2}, \pm\sqrt{10}\}$

Solve $x^{2/5} - 3x^{1/5} + 2 = 0$

Hint: Let $u = x^{1/5}$

$u^2 - 3u + 2 = 0$

$u^2 = (x^{1/5})^2 = x^{2/5}$

$(u-2)(u-1) = 0$

$u-2=0$ $\quad u-1=0$

$u=2$ $\quad u=1$

when $u=2$

$x^{1/5} = 2$

$(\sqrt[5]{x})^5 = (2)^5$

$x=32$

when $u=1$

$x^{1/5} = 1$

$(\sqrt[5]{x})^5 = (1)^5$

$x=1$

$\{1, 32\}$

Parabola

$y = a(x-h)^2 + K$

$a \neq 0$

Vertex (h,K)

$a>0$

$a<0$

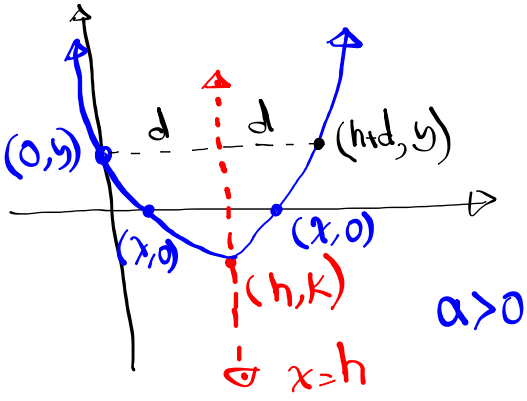
Vertex (h,K)

Axis of Symmetry

$x = h$

Y-Int → Let $x=0$ → find y

x-Int → Let $y=0$ → find x



Consider $y = \frac{1}{2}(x+2)^2 + 3$

$y = a(x-h)^2 + K$

$a = \frac{1}{2}$ $a>0$, opens upward

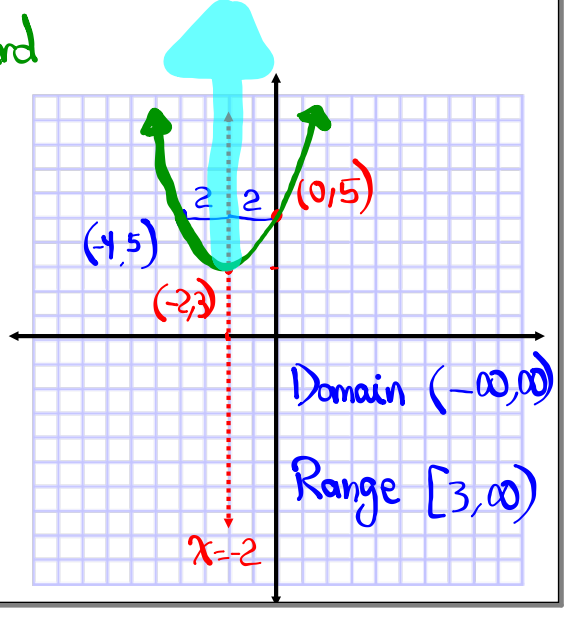
$h = -2$ → Vertex $(-2, 3)$

$K = 3$

A.O.S. $x = -2$

Y-Int → $x=0$ → $(0,5)$

x-Int → None



Consider $y = -(x+3)^2$ 0

$y = a(x-h)^2 + k$

$a = -1$

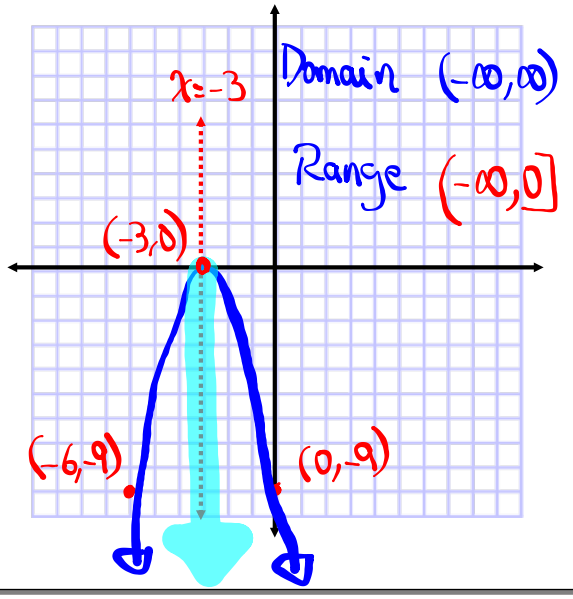
Direction opens down

Vertex $(-3, 0)$

A.O.S. $x = -3$

Y-Int $(0, -9)$

X-Ints $(-3, 0)$



Consider $y = -x^2 + 4$ $a = -1$

$y = -(x-0)^2 + 4$ Direction opens down

$y = a(x-h)^2 + k$ Vertex $(0, 4)$

A.O.S. $x = 0$

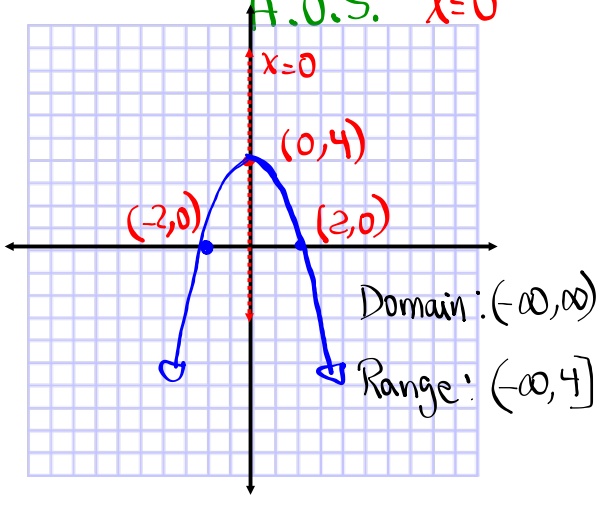
Y-Int $(0, 4)$

X-Int $\rightarrow y = 0$ $(2, 0)$

$-x^2 + 4 = 0$ $(-2, 0)$

$-x^2 = -4$

$x^2 = 4$ $\boxed{x = \pm 2}$



Ex: $y = (x-3)^2 - 4$

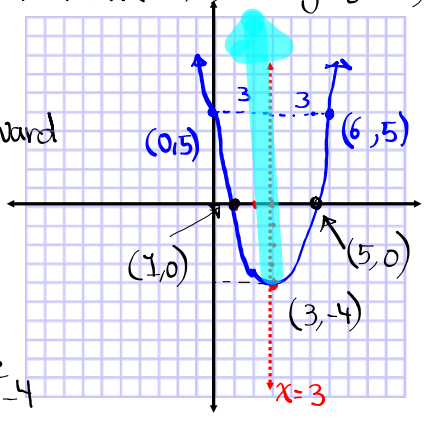
$y = a(x-h)^2 + k$

Domain: $(-\infty, \infty)$ Range: $[-4, \infty)$

$a = 1$ $a > 0 \rightarrow$ opens upward

$h = 3$ $k = -4$ Vertex $(3, -4)$

A.O.S. $x = 3$



Y-Int $\rightarrow x = 0 \rightarrow y = (0-3)^2 - 4$

$= 9 - 4$

$= 5$

X-Int $\rightarrow y = 0 \rightarrow (x-3)^2 - 4 = 0$

$(x-3)^2 = 4$

$x - 3 = \pm\sqrt{4}$

$x = 3 \pm 2$

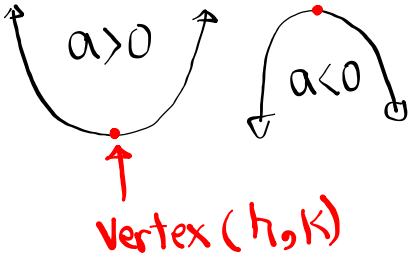
$x = 5 \rightarrow (5, 0)$

$x = 1 \rightarrow (1, 0)$

Parabola

$y = ax^2 + bx + c$

$a \neq 0$



Vertex (h, k)

Vertex (h, k)

$h = \frac{-b}{2a}$

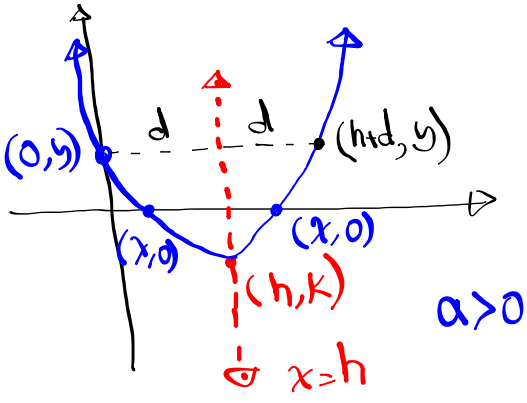
$k \rightarrow$ Plug in h, to find k.

Axis of Symmetry

$x = h$

Y-Int $\rightarrow$ Let $x = 0 \rightarrow$ find y

X-Int $\rightarrow$ Let $y = 0 \rightarrow$ find x



Ex: $y = x^2 - 2x + 5$

$y = ax^2 + bx + c$

$a = 1 \quad b = -2 \quad c = 5$

$a > 0 \rightarrow$ opens up

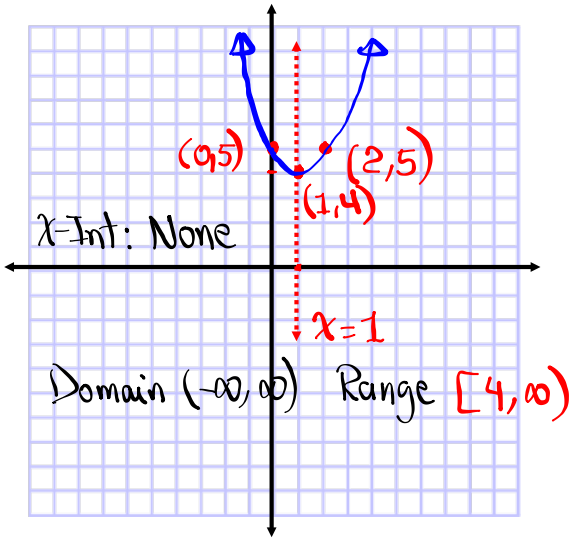
$h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$k = (1)^2 - 2(1) + 5 = 4$

Vertex $(1, 4)$

A.O.S. $x = 1$

Y-Int $(0, 5)$



Consider $y = -x^2 - 6x$

$y = ax^2 + bx + c$

$a = -1$ opens down

$b = -6$

$c = 0$

$h = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3 \Rightarrow$ Vertex $(-3, 9)$

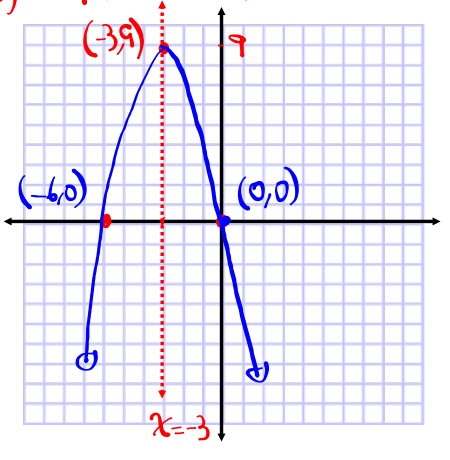
$k = -(-3)^2 - 6(-3) = -9 + 18 = 9$ A.O.S. $x = -3$

Y-Int $(0, 0)$

x-Int $(0, 0), (-6, 0)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 9]$



Class QZ 20

Solve $(\sqrt{x}-1)^2 - 5(\sqrt{x}-1) + 6 = 0$

Let $u = \sqrt{x} - 1$

$u^2 - 5u + 6 = 0$

$(u-2)(u-3) = 0$

$u = 2 \quad u = 3$

when $u = 2$: $\sqrt{x} - 1 = 2$, $\sqrt{x} = 3$, $x = 9$

when $u = 3$: $\sqrt{x} - 1 = 3$, $\sqrt{x} = 4$, $x = 16$

$\{9, 16\}$